Nomenclature and numerical standards for IAU models and IERS Conventions for Earth rotation

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Introduction

- The recent IAU (2000 and 2006) resolutions on reference systems and Earth rotation include:
  - a change in the way the celestial orientation of the Earth is expressed,
  - a new nomenclature associated with the new concepts,
  - new models for quantities necessary for expressing Earth rotation.

- The aim of this presentation is to review the changes for IAU models and IERS Conventions resulting from the joint IAU 2000/2006 resolutions on:
  - the new nomenclature, parameters and models,
  - the status and numerical values of the constants associated with the IAU models for precession, nutation and the angle for Earth rotation with respect to those of the current (1976) IAU system of astronomical constants and previous lists of Current Best estimates.

- SOFA, which provides the authoritative algorithms for implementing the IAU resolution, will be considered in the next presentation in session 2.

The IAU 2000/2006 Resolutions for Earth rotation

IAU 2000 Resolutions

Resolution B1.3

Definition of BCRS and GCRS

Barycentric celestial reference system (BCRS)

Geocentric celestial reference system (GCRS)

Resolution B1.6

IAU 2000 Precession-Nutation Model

Resolution B1.7

Definition of Celestial Intermediate Pole

Resolution B1.8

Definition and use of CEO and TEO

ERA(UT1)=2π (0.7790572732640+1.00273781191135448 x (Julian UT1date-2451545.0)

Resolution B1.9

Re-definition of TT as a linear transformation of TCG

IAU 2006 Resolutions

Resolution B1

Adoption of the P03 Precession and definition of the ecliptic

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Resolution B2

Supplement to the IAU 2000 Resolutions on reference systems

Rec 1: *Harrmonizing « intermediate » to the pole and the origin* + definition of celestial intermediate (celestial/terrestrial) systems

Rec 2: *Default orientation of the BCRS and GCRS*

Resolution B3

Re-definition of TDB as a linear transformation of TCB

Nomenclature for Earth rotation

A few examples from the NFA IAU 2006 Glossary (of the IAU WG Nomenclature in Fundamental Astronomy) available at http://syrte.obspm.fr/iauWGnfa/

Celestial Intermediate Origin (CIO)

origin for right ascension on the intermediate equator in the celestial intermediate reference system. It is the non-rotating origin in the GCRS that is recommended by the IAU 2000 Resolution B 1.8, where it was designated the Celestial Ephemeris Origin. The CIO was originally set close to the GCRS meridian and throughout 1900-2100 stays within 0.1 arcseconds of this alignment.

Celestial Intermediate Pole (CIP)

geocentric equatorial pole defined by IAU 2000 Resolution B1.7 as being the intermediate pole, in the transformation from the GCRS to the ITRS, separating nutation from polar motion. It replaced the CEP on 1 January 2003. Its GCRS position results from (i) the part of precession-nutation with periods greater than 2 days, and (ii) the retrograde diurnal part of polar motion (including the free core nutation, FCN) and (iii) the frame bias. Its ITRS position results from (i) the part of polar motion which is outside the retrograde diurnal band in the ITRS and (ii) the motion in the ITRS corresponding to nutations with periods less than 2 days. The motion of the CIP is realized by the IAU precession-nutation plus time-dependent corrections provided by the IERS.

Earth Rotation Angle (ERA)

angle measured along the intermediate equator of the Celestial Intermediate Pole (CIP) between the Terrestrial Intermediate Origin (TIO) and the Celestial Intermediate Origin (CIO), positively in the retrograde direction. It is related to UT1 by a conventionally adopted expression in which ERA is a linear function of UT1 (see IAU 2000 Resolution B1.8). Its time derivative is the Earth's angular velocity. Previously, it has been referred to as the stellar angle.

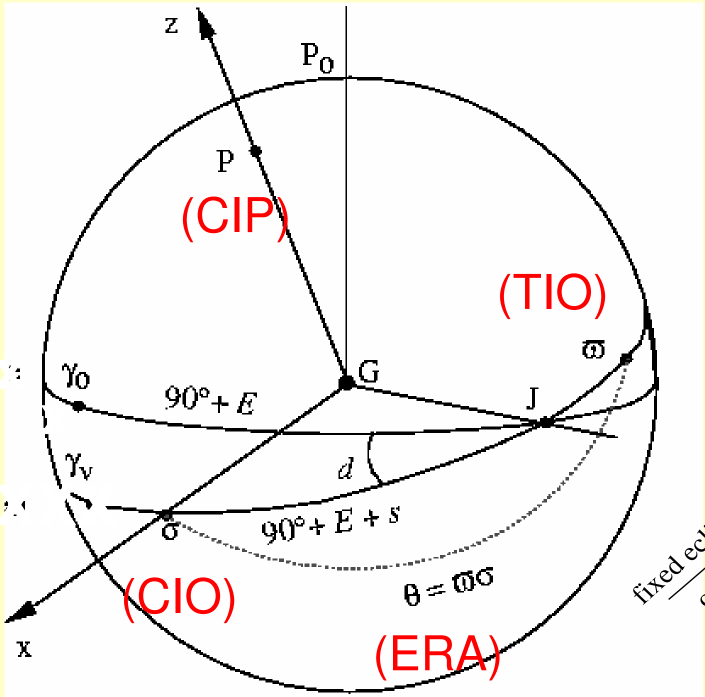
Celestial Intermediate Reference System (CIRS)

geocentric reference system related to the GCRS by a time-dependent rotation taking into account precession-nutation. It is defined by the intermediate equator (of the CIP) and CIO on a specific date (IAU 2006 Resolution B2). It is similar to the system based on the true equator and equinox of date, but the equatorial origin is at the CIO. Since the acronym for this system is close to another acronym (namely ICRS), it is suggested that wherever possible the complete name is used.

Parameters for Earth rotation

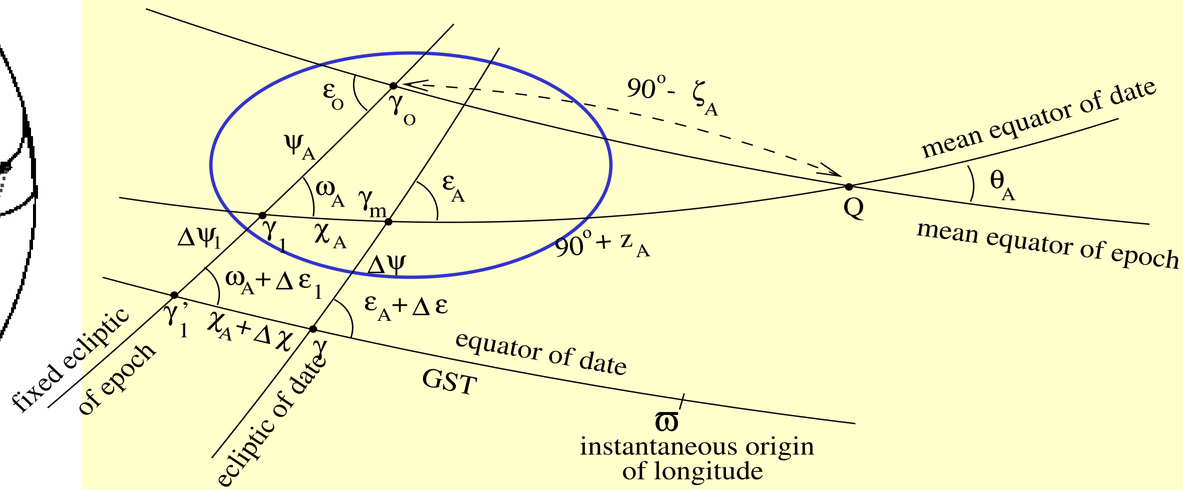
The GCRS coordinates X, Y of the CIP unit vector,

the Earth rotation angle, ERA,



replaces

the classical precession and nutation angles
Greenwich sidereal time, GST



Models for Earth rotation

IAU Precession-nutation

1) IAU 2000 Resolution B1.6

- adopted the IAU2000 precession-nutation (Mathews, Herring, Buffett 2002) which was implemented in the IERS Conventions 2003

IAU 2000A Nutation *(non-rigid Earth)*

IAU 2000 Precession= IAU 1976 *(Lieske et al. 1977)* + corrections to precession rates

$d\psi_A$ (IAU 2000) = −0.299 65"/c ; $d\omega_A$ (IAU 2000) = −0.025 24"/c

1st step

Celestial pole offsets *at J2000 (VLBI estimates)*

ξ_0 (IAU 2000) = −16.6170 mas ; η_0 (IAU 2000) = −6.8192 mas

- recommended the development of new expressions for precession consistent with dynamical theories and with IAU 2000A nutation

2) IAU 2006 Resolution B1

- adopted the P03 precession (Capitaine, Chapront, Wallace, 2003) dynamical model consistent with IAU 2000A nutation and with non-rigid Earth (Hilton et al. 2006)

2nd step

- recommended improved definitions (ecliptic, precession of the equator, precession of the ecliptic)

Transformation between the ITRS and GCRS in the IERS Conventions (Chapter 5)

The coordinate transformation from the terrestrial system, ITRS to the Geocentric celestial system, GCRS, can be written as:

$$[GCRS] = Q(t)\ R(t)\ W(t)\ [ITRS]$$

rotation angle
(model + observed variations)
IAU 2000 IERS

celestial motion of the CIP
(model + observed celestial pole offsets
IAU 2000/2006 IERS

terrestrial motion of the CIP
(observed pole coordinates)
IERS

Celestial | | *terrestrial*
intermediate reference system

Numerical standards for Earth rotation

1st system of fundamental astronomical constants

(Conférence internationale des étoiles fondamentales, Paris, 1896)

1° La *constante de la précession* . . . . $p = 50'',2564$;

2° La *constante de la nutation* . . $N_0 = 9'',21$;

3° *L'obliquité de l'écliptique* . . . $\varepsilon = 23°27'8'',26$;

4° La *constante de l'aberration* . . $\kappa = 20'',47$;

5° La *parallaxe solaire* . . . . $\varpi_0 = 8'',80$.

6° *L'aplatissement de l'ellipsoïde terrestre* . . . . $\alpha = 1/297$;

7° Le *rayon équatorial terrestre* . . . . $a_e = 6\,378\,388$ m;

8° La *constante de la gravitation universelle* . . . . $k = 0,017\,202\,098\,95$.

3 constants (p, N_0 and ε) associated with precession/nutation

IAU 1976 System of astronomical constants

Defining constants:

1.	Gaussian gravitational constant	$k = 0{\cdot}017\ 202\ 098\ 95$
2.	Speed of light	$c = 299\ 792\ 458$ m s^{-1}

Primary constants:

3.	Light-time for unit distance	$\tau_A = 499{\cdot}004\ 782$ s [499·004 7838 ...]
4.	Equatorial radius for Earth	$a_e = 6378\ 140$ m [6378 137]
5.	Dynamical form-factor for Earth	$J_2 = 0{\cdot}001\ 082\ 63$ [0·001 082 626]
6.	Geocentric gravitational constant	$GE = 3{\cdot}986\ 005 \times 10^{14}$ m^3 s^{-2} [$3{\cdot}986\ 004\ 33 \ldots \times 10^{14}$]
7.	Constant of gravitation	$G = 6{\cdot}672 \times 10^{-11}$ m^3 kg^{-1} s^{-2}
8.	Ratio of mass of Moon to that of Earth	$\mu = 0{\cdot}012\ 300\ 02$ [0·012 300 038]
9.	General precession in longitude, per Julian century, at standard epoch 2000	$\rho = 5029''0966$
10.	Obliquity of the ecliptic, at standard epoch 2000	$\epsilon = 23°\ 26'\ 21''448$

Derived constants:

11.	Constant of nutation, at standard epoch 2000	$N = 9''2025$
12.	Unit distance	$c\tau_A = A = 1{\cdot}495\ 978\ 70 \times 10^{11}$ m [$1{\cdot}495\ 978\ 706\ 91 \times 10^{11}$]
13.	Solar parallax	$\arcsin(a_e/A) = \pi_\odot = 8''794\ 148$ [8''794 144]
14.	Constant of aberration, for standard epoch 2000	$\kappa = 20''49\ 552$
15.	Flattening factor for the Earth	$f = 0{\cdot}003\ 352\ 81$ $= 1/298{\cdot}257$
16.	Heliocentric gravitational constant	$A^3k^2/D^2 = GS = 1{\cdot}327\ 124\ 38 \times 10^{20}$ m^3 s^{-2} [$1{\cdot}327\ 124\ 40 \ldots \times 10^{20}$]
17.	Ratio of mass of Sun to that of the Earth	$(GS)/(GE) = S/E = 332\ 946{\cdot}0$ [332 946·050 895 ...]
18.	Ratio of mass of Sun to that of Earth + Moon	$(S/E)/(1+\mu) = 328\ 900{\cdot}5$ [328 900·561 400]
19.	Mass of the Sun	$(GS)/G = S = 1{\cdot}9891 \times 10^{30}$ kg

3 constants (p, ε and N) associated with precession/nutation with updated values

The Numerical Standards of the IERS Conventions (2003)

Table 1.1 IERS Numerical Standards.

ITEM	VALUE	UNCERTAINTY	REF.	COMMENTS
c	$299792458 ms^{-1}$	Defining	[2]	Speed of light
L_B	$1.55051976772 \times 10^{-8}$	2×10^{-17}	[4]	Average value of 1-d(TT)/d(TCB)
L_C	$1.48082686741 \times 10^{-8}$	2×10^{-17}	[4]	Average value of 1-d(TCG)/d(TCB)
L_G	$6.969290134 \times 10^{-10}$	Defining	[4]	1-d(TT)/d(TCG)
G	$6.673 \times 10^{-11} m^3 kg^{-1} s^{-2}$	$1 \times 10^{-13} m^3 kg^{-1} s^{-2}$	[2]	Constant of gravitation
$GM_\odot$	$1.32712442076 \times 10^{20} m^3 s^{-2}$	$5 \times 10^{10} m^3 s^{-2}$	[from 3]	Heliocentric gravitational constant
$\tau_A{}^\dagger$	$499.0047838061 s$	$0.00000002 s$	[3]	Astronomical unit in seconds
$c\tau_A{}^\dagger$	$149597870691 m$	$6 m$	[3]	Astronomical unit in meters
$\psi_1{}^\dagger$	$5038.47875''/c$	$0.00040''/c$	[6]	IAU(1976) value of precession of the equator at J2000.0 corrected by $-0.29965''$. See Chapter 5.
ϵ_0	$84381.4059''$	$0.0003''$	[5]	Obliquity of the ecliptic at J2000.0. See Chapter 5 for value used in IAU precession-nutation model.
$J_{2\odot}$	2×10^{-7}	(adopted for DE405)		Dynamical form-factor of the Sun
μ	0.0123000383	5×10^{-10}	[3]	Moon-Earth mass ratio
$GM_\oplus$	$3.986004418 \times 10^{14} m^3 s^{-2}$	$8 \times 10^5 m^3 s^{-2}$	[1]	Geocentric gravitational constant (EGM96 value)
$a_E{}^\ddagger$	$6378136.6 m$	$0.10 m$	[1]	Equatorial radius of the Earth
$1/f^\ddagger$	298.25642	0.00001	[1]	Flattening factor of the Earth
$J_{2\oplus}{}^\ddagger$	1.0826359×10^{-3}	1.0×10^{-10}	[1]	Dynamical form-factor
ω	$7.292115 \times 10^{-5} rad s^{-1}$	variable	[1]	Nominal mean angular velocity of the Earth
$g_e{}^\ddagger$	$9.7803278 ms^{-2}$	$1 \times 10^{-6} ms^{-2}$	[1]	Mean equatorial gravity
W_0	$62636856.0 m^2 s^{-2}$	$0.5 m^2 s^{-2}$	[1]	Potential of the geoid
$R_0{}^{\dagger\dagger}$	$6363672.6 m$	$0.1 m$	[1]	Geopotential scale factor

† The values for τ_A, $c\tau_A$, and ψ_1 are given in "TDB" units (see discussion above).

‡ The values for a_E, $1/f$, $J_{2\oplus}$ and g_E are "zero tide" values (see the discussion in section 1.1 above). Values according to other conventions may be found from reference [1].

†† $R_0 = GM_\oplus / W_0$

[1] Groten, E., 1999, Report of the IAG. Special Commission SC3, Fundamental Constants, XXII IAG General Assembly.

[2] Mohr, P. J. and Taylor, B. N., 1999, *J. Phys. Chem. Ref. Data*, **28**, **6**, p. 1713.

[3] Standish, E. M., 1998, JPL IOM 312-F.

[4] IAU XXIV General Assembly. See Appendix 1.

[5] Fukushima, T., 2003, Report on astronomical constants, *Highlights of Astronomy*, in press.

[6] Mathews, P. M., Herring, T. A., and Buffett, B. A., 2002, Modeling of nutation-precession: New nutation series for nonrigid Earth, and insights into the Earth's interior, *J. Geophys. Res.* **107**, **B4**, 10.1029/2001JB00390.

2 constants (ε_0 and ψ_1) associated with precession
→ change of status for p

0 constant for nutation
→ change of status

+ 1 constant (ω) associated with Earth rotation

Main features of the IAU 2000 nutation

Rigid Earth nutation → MHB2000 nutation

analytical solution (Souchay et al. 1999) *transfer function* *(Mathews et al. 2002)*

semi-analytical series: 1365 luni-solar and planetary terms

Transfer function

derived from the solution of equations obtained by generalization of the *SOS equations* (Sasao et al. 1980) with the MHB2000 *Basic Earth Parameters* fitted to VLBI data.

Basic Earth Parameters	Estimate	Correction to hydrostatic equilibrium
e_f	0.0026456 ±20	0.0000973
κ	0.0010340 ±92	-0.0000043
γ	0.0019662 ±14	0.0000007
e	0.0032845479 ±12	0.000037
Im $K^{(CMB)}$	-0.0000185 ±14	
Re $K^{(ICB)}$	0.00111 ±10	
Im $K^{(ICB)}$	-0.00078 ±13	
rms residuals	0.0132 mas	

+ obliquity rate fitted to VLBI

Essential MHB2000 parameters

- Earth's dynamical flattening $\mathbf{H_d=e/(1+e)}$: scale factor for the precession rate and nutation amplitudes,
- dynamical flattening of the core, e_f : resonance factor for the nutation amplitudes.

The numerical standards associated with the IAU 2000 nutation

The numerical standards/constants associated with the IAU 2000 nutation could be:

- the numerical values for the nutation amplitudes,
- the numerical values for the fundamental MHB2000 Basic Earth parameters (e.g. ***e* and *e*$_f$**).

Mean features of the IAU 2006 precession

The precession of the equator is solution of the dynamical precession equations , with contributions to the precession rates r_ψ , r_ε (from Souchay et al. 1990/1999, Williams 1994, Brumberg et al. 1998, Mathews et al. 2002)

Source of the effect	ϵ dependence	Contribution in longitude at J2000			Contribution in obliquity at J2000		
		μas/cy	μas/cy^2	μas/cy^3	μas/cy	μas/cy^2	μas/cy^3
Luni-solar & Planetary torque							
Luni-solar 1st order	$\cos\epsilon$	$(r_0)_1$	-3395	-6	0	0	0
Luni-solar 2d order(a)	$6\cos^2\epsilon - 1$	$-33\,100$	0	0	0	0	0
Luni-solar 2d order(b)	$3\cos^2\epsilon - 1$	$-13\,680$	0	0	0	0	0
Luni-solar J_4	$\cos\epsilon\left(4 - 7\sin^2\epsilon\right)$	$+2600$	0	0	0	0	0
Planetary 1st order	$\cos\epsilon$	$+31\,367$	0	0	-1400	0	0
J_2 and planetary tilts							
J_2 and planetary tilt(a)	$\cos 2\epsilon/\sin\epsilon$	$-269\,430$	$+1074$	0	0	0	0
planetary tilt(b)	$\cos\epsilon$	0	0	0	$(u_0)_1$	-44	$+3$
Tides							
tides(a)	$\cos^2\epsilon$	0	-102	0	0	0	0
tides(b)	$\cos^3\epsilon$	0	-133	0	0	0	0
tides(c)	$\sin\epsilon\cos\epsilon$	0	0	0	$+2400$	0	0
J_2 rate	$\cos\epsilon$	0	$-14\,000$	0	0	0	0
Non-linear effect	1	$-21\,050$	0	0	0	0	0
Geodesic precession	1	$-1\,919\,883$	$+3$	$+1$	-1	0	$+5$
Total		$(r_0)_1 - 2\,223\,176$	$-16\,553$	-5	$(u_0)_1 + 999$	-44	$+8$

$(r_0)_1$ and $(u_0)_1$ derived from VLBI observations

+ correction to the precession rate for the change in the J2000 obliquity from IAU2000 to P03

+ J_2 rate value: dJ_2/dt :

$-$ 3.0 x10^{-11}/ yr

Physical parameters: dynamical flattening, H_d and J_2 rate value

IAU 2000: $H_d = 3.273\,794\,92\ 10^{-3}$

IAU 2006: $H_d = 3.273\,794\,48\ 10^{-3}$

P03 expressions for the fundamental precession parameters

(Capitaine et al. 2003)

Ecliptic / *Equator*

Source		t^0	t	t^2	t^3	t^4	t^5
IAU 2000	P_A		4197.6	194.47	−0.179		
P03			4199.094	193.9873	−0.22466	−0.000912	0.0000120
IAU	Q_A		−46815.0	50.59	0.344		
P03			−46811.015	51.0283	0.52413	−0.000646	−0.0000172
IAU 2000	ψ_A		5038478.750	−1072.59	−1.147		
P03			5038481.507	−1079.0069	−1.14045	0.132851	−0.0000951
IAU 2000	ω_A	84381448.0	−25.240	51.27	−7.726		
P03		84381406.0	−25.754	51.2623	−7.72503	−0.000467	0.0003337

CIO based precession quantities

Source	t^0	t	t^2	t^3	t^4	t^5
X	− 16.617	2004191.898	− 429.7829	−198.61834	0.007578	0.0059285
Y	− 6.951	−25.896	− 22407.2747	1.90059	1.112526	0.0001358
$s+XY/2$	0.094	3.80865	− 0.12268	− 72.57411	0.02798	0.01562

ψ_{A1}, ω_{A1}, X_1, Y_1: precession rates for the equator → change of status for the precession rate

Expressions for the IAU 2006 precession

TABLE I
The polynomial coefficients for the precession angles.

Angle	(arcsec.)	Coefficients* $\left(\frac{\text{arcsec.}}{\text{cent.}}\right)$	$\left(\frac{\text{arcsec.}}{\text{cent.}^2}\right)$	$\left(\frac{\text{arcsec.}}{\text{cent.}^3}\right)$	$\left(\frac{\text{arcsec.}}{\text{cent.}^4}\right)$	$\left(\frac{\text{arcsec.}}{\text{cent.}^5}\right)$
ψ_A		5038.481507	−1.0790069	−0.00114045	0.000132851	-9.51×10^{-8}
ω_A	84381.406000	−0.025754	0.0512623	−0.00772503	-4.67×10^{-7}	3.337×10^{-7}
P_A		4.199094	0.1939873	−0.00022466	-9.12×10^{-7}	1.20×10^{-8}
Q_A		−46.811015	0.0510283	0.00052413	-6.46×10^{-7}	-1.72×10^{-8}
π_A		46.998973	−0.0334926	−0.00012559	1.13×10^{-7}	-2.2×10^{-9}
Π_A	629546.7936	−867.95758	0.157992	−0.0005371	−0.00004797	7.2×10^{-8}
ϵ_A †	84381.406000	−46.836769	−0.0001831	0.00200340	-5.76×10^{-7}	-4.34×10^{-8}
χ_A		10.556403	−2.3814292	−0.00121197	0.000170663	-5.60×10^{-8}
z_A	−2.650545	2306.077181	1.0927348	0.01826837	−0.000028596	-2.904×10^{-7}
ζ_A	2.650545	2306.083227	0.2988499	0.01801828	-5.971×10^{-6}	-3.173×10^{-7}
θ_A		2004.191903	−0.4294934	−0.04182264	-7.089×10^{-6}	-1.274×10^{-7}
p_A		5028.796195	1.1054348	0.00007964	−0.000023857	3.83×10^{-8}
X	−0.016617	2004.191898	−0.4297829	−0.19861834	7.578×10^{-6}	5.9285×10^{-6}
Y	−0.006951	−0.025896	−22.4072747	0.00190059	0.001112526	1.358×10^{-7}
$s + \frac{1}{2}XY$	0.0000940	0.00380865	−0.00012268	−0.07257411	0.00002798	0.00001562
γ_{J2000}		10.556403	0.4932044	−0.00031238	-2.788×10^{-6}	2.60×10^{-8}
ϕ_{J2000}	84381.406000	−46.811015	0.0511269	0.00053289	-4.40×10^{-7}	-1.76×10^{-8}
ψ_{J2000}		5038.481507	1.5584176	−0.00018522	−0.000026452	-1.48×10^{-8}
γ_{GCRS}	−0.052928	10.556378	0.4932044	−0.00031238	-2.788×10^{-6}	2.60×10^{-8}
ϕ_{GCRS}	84381.412819	−46.811016	0.0511268	0.00053289	-4.40×10^{-7}	-1.76×10^{-8}
ψ_{GCRS}	−0.041775	5038.481484	1.5584175	−0.00018522	−0.000026452	-1.48×10^{-8}

*Centuries (cent.) are Julian centuries of 36,525 days TT.
†The angle $\epsilon_0 \equiv \epsilon_A(t=0)$.

(Hilton et al. 2006)

The numerical standards associated with the IAU 2006 precession

The numerical standards/constants associated with the IAU 2006 precession could be:

- the numerical values for the coefficients of the polynomial expressions for the precession quantities,
- the numerical values for the fundamental parameters for precession (e.g. H_d and J_2 rate).

Numerical standards associated with the Earth rotation

- ***Earth Rotation Angle***

ERA = $\theta = k$ UT1 ; $d\theta/dt = \omega_3$

$$\mathrm{ERA(UT1)} = 2\pi\,(0.7790572732640 + 1.00273781191135448 \times (\text{Julian UT1date} - 2451545.0))$$

ERA(UT1) : conventional relationship that defines UT1 from ERA,

0.7790572732640 and 1.00273781191135448 rev/day are defining constants.

(which replaces the classical factor between GMST and UT1).

*(*The ERA/UT1 relationship involves not only the Earth angular velocity but also what UT1 is intended to represent, i.e. the hour angle of the « ficticious mean » Sun.)

- ***Nominal mean angular velocity of the Earth***

$\omega = 7.292\,115\ 10^{-5}$ rad /s

this value has been chosen to have the number of significant digits limited to those for which the value can be considered a constant; the value is "significantly" different from the actual value of omega as there are relative variations in omega of the order of 10^{-7}.

The choice of numerical standards

- ***Nutation***

Numerical values for the nutation amplitudes or for the fundamental MHB2000 Basic Earth parameters ?

too much necessary constants → model

- ***Precession***

Numerical values for the coefficients of the polynomial expressions for the precession quantities or for the fundamental parameters for precession.

too much necessary constants → list of expressions

- ***Earth rotation***

- 2 numerical constants for the conventional relationship that defines UT1 from ERA,

- 1 numerical constant for the nominal mean angular velocity of the Earth

Summary

- The IAU 2000/2006 Resolutions have adopted new concepts, new nomenclature and new models for Earth rotation, which have modified the status and numerical values of the constants associated with Earth rotation.

- The high accuracy observations of Earth rotation requires a coordinated use of numerical standards and IAU-approved formulations and software that implement the IAU models.

- A few constants are no more sufficient for representing precession and nutation but should be replaced by the precession-nutation models.

- A very few numerical constants are sufficient for the Earth's rotation.